

S. S. College. Jehanabad (Magadh University)

Department : Physics

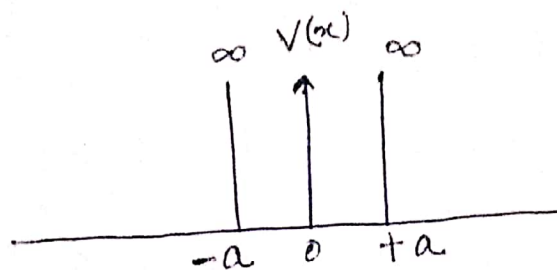
Subject : Quantum Mechanics

Class : B.Sc(H) Physics Part III

Topic: One dimensional infinite potential square well.

Teacher : M. K. Singh

1. one dimensional infinite square well



Let us consider a particle of mass 'm' confined in region of width $2a$ from $x = -a$ to $x = a$. The particle at $x = \pm a$ experiences large force. Therefore, the potential energy

$$V(x) = \begin{cases} 0 & |x| < a \\ \infty & |x| > a \end{cases} \quad - (1)$$

The time independent schrodinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad - (2)$$

Since potential is infinite at $x = \pm a$, the probability of finding the particle outside the well is zero.

$$\text{for } |x| > a \quad \psi(x) = 0 \quad \text{at } x = \pm a \quad - (3)$$

For $|x| < a$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2} \psi \quad - (4)$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0 \Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad - (5)$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

The general solution is given by

$$\psi(x) = A \sin kx + B \cos kx \quad - (6)$$

Applying the boundary condition; $x = a$

$$A \sin ka + B \cos ka = 0 \quad - (7)$$

and at $x = -a$

$$-A \sin ka + B \cos ka = 0 \quad - (8)$$

from Eq (7) & (8)

$$A \sin ka = 0 \quad \& \quad B \cos ka = 0$$

We cannot allow, $A = B = 0$ (trivial solution)

The two possible solutions are

$$\left. \begin{array}{l} A = 0 \quad \& \quad \cos ka = 0 \\ \text{or} \\ B = 0 \quad \& \quad \sin ka = 0 \end{array} \right\} \Rightarrow ka = n\pi/2$$

Thus the eigenfunctions are

$$\psi_n(x) = B \cos\left(\frac{n\pi x}{2a}\right) \quad \text{or}$$

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{2a}\right)$$

In order to normalize,

$$\int_{-a}^a \psi_n^*(x) \psi_n(x) dx = 1$$

$$A^2 \int_{-a}^a \sin^2\left(\frac{n\pi x}{2a}\right) dx = B^2 \int_{-a}^a \cos^2\left(\frac{n\pi x}{2a}\right) dx = 1$$

$$A = B = \frac{1}{\sqrt{a}}$$

$$\psi_n(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{n\pi x}{2a}\right); \quad n = 1, 3, 5, \dots$$

$$\psi_n(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi x}{2a}\right); \quad n = 2, 4, 6, \dots$$

The energy eigenvalue, $E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{8ma^2}; \quad n = 1, 2, 3, \dots$